LaVision
FOCUS ON IMAGING

Durham University

In Application

Obtaining Maps of Elastic Constants in a Multi-Material Component

S. Szyniszewski et al, Department of Engineering, Durham University, UK

Introduction

Understanding the spatial distribution of material stiffness provides important insight into as-manufactured properties. This is particularly relevant in applications where additive manufacturing and other manufacturing processes exhibit inherent variability, resulting in a degree of uncertainty in the final product quality due to random factors such as humidity, temperature, and natural variations in the properties of the raw materials. Such variability can introduce weak regions and/or localized soft spots, potentially compromising reliability in safety-critical applications such as bespoke medical implants, on-demand 3D-printed aerospace replacement parts, and soft robotic components, among others.

In this study, we present a numerical algorithm for computing spatial maps of material stiffness across a specimen. The method uses full-field strain measurements obtained through digital image correlation (DIC), together with the specimen geometry and applied loading, to determine local elastic constants by solving the underlying mechanical problem while enforcing equilibrium and compatibility. The use of DIC for characterizing spatially inhomogeneous material properties offers considerable advantages over conventional techniques, such as nanoindentation performed over a grid of discrete points on the specimen surface.

Experimental Setup

The benchmark plate comprising two distinct material formulations with different stiffnesses, arranged in a chessboard-like pattern, was tested to validate the computational approach. The specimen was 3D-printed with 15 mm thickness using two carefully controlled ratios of cross-linking agent to base material, with each ratio producing a distinct level of mechanical stiffness. The two formulations were assigned to alternating square regions across the plate, such that adjacent regions generally contained different formulations, thereby producing a regular pattern of step changes in stiffness throughout the benchmark sample. The prescribed spatial distribution of material stiffness is shown in Figures 1 and 2.

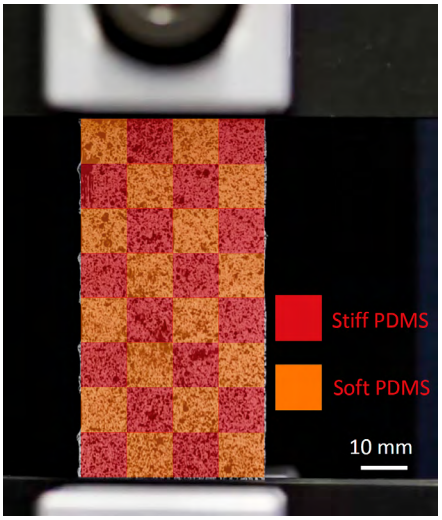


Figure 1: *Experimental testing setup of two-phase (soft and stiff) PDMS composite.*

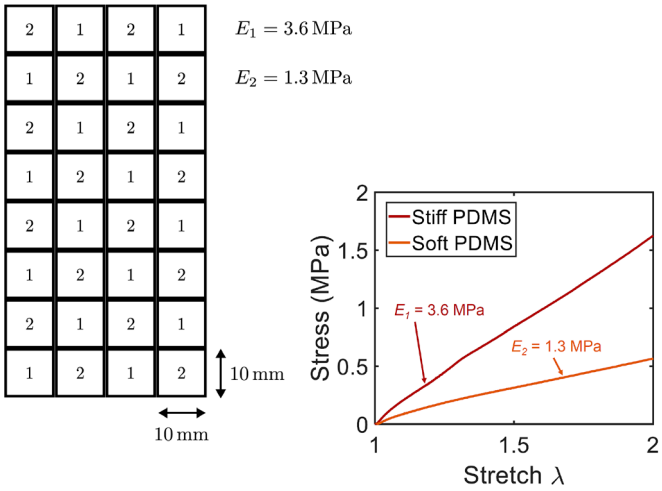


Figure 2: *Material pattern of soft and stiff PDMS materials, and their respective properties.*

LaVisionUK Ltd
2 Minton Place / Victoria Road
Bicester, Oxon / OX26 6QB / United Kingdom
E-Mail: sales@lavision.com / www.lavisionuk.com
Phone: +44-(0)-870-997-6532 / Fax: +44-(0)-870-762-6252

LaVision GmbH
Anna-Vandenhoeck-Ring 19
37081 Göttingen / Germany
E-Mail: info@lavision.com / www.lavision.com
Tel. +49-(0)551-9004-0 / Fax +49-(0)551-9004-100

LaVision Inc.
211 W. Michigan Ave. / Suite 100
Ypsilanti, MI 48197 / USA
E-mail: sales@lavisioninc.com / www.lavisioninc.com
Phone: (734) 485 - 0913 / Fax: (240) 465 - 4306

Obtaining Maps of Elastic Constants in a Multi-Material Component

S. Szyniszewski et al, Department of Engineering, Durham University, UK

Results

The raw images acquired using the DIC setup were processed using LaVision's DaVis 11 software to generate full-field strain maps. The processing was done with 31 x 31 pix subsets and 10 pix overlap. The final strain map consisted of 47 elements x 91 elements = 4277 elements.

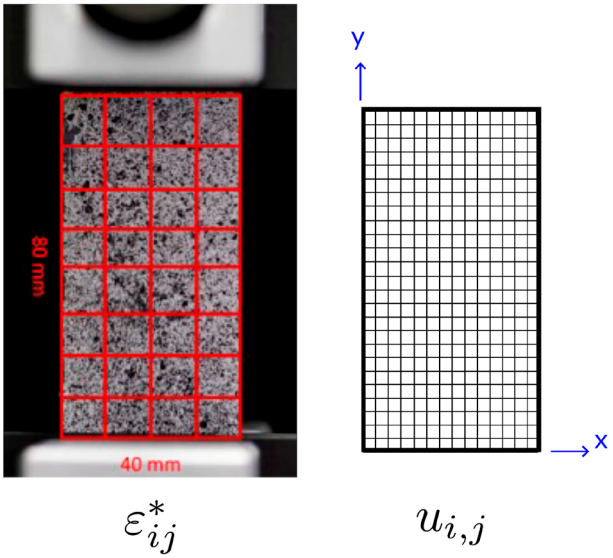


Figure 3: *Digital image correlation pattern and a corresponding finite element mesh resolution of an accompanying computational model.*

DIC measurements were subsequently supplemented with a computational model with a matching 47 x 91 mesh resolution, where the DIC-derived strain fields were then used as input to an iterative computational algorithm, together with information on the applied loading, overall force equilibrium. The tensile load applied was 360 kN. Figure 4 shows the strain calculated directly from the DIC displacements, along with the strains matched via the further analysis described below.

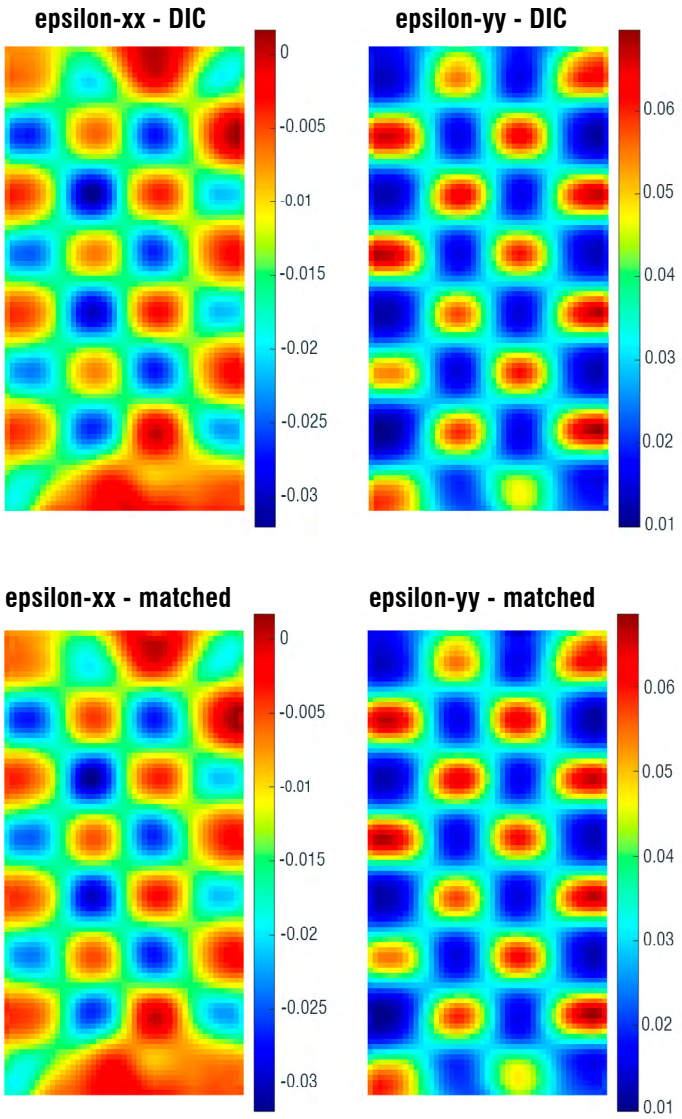


Figure 4: *DIC (input) and matched strain maps after 10 iterations.*

LaVisionUK Ltd
2 Minton Place / Victoria Road
Bicester, Oxon / OX26 6QB / United Kingdom
E-Mail: sales@lavision.com / www.lavisionuk.com
Phone: +44-(0)-870-997-6532 / Fax: +44-(0)-870-762-6252

LaVision GmbH
Anna-Vandenhoeck-Ring 19
37081 Göttingen / Germany
E-Mail: info@lavision.com / www.lavision.com
Tel. +49-(0)551-9004-0 / Fax +49-(0)551-9004-100

LaVision Inc.
211 W. Michigan Ave. / Suite 100
Ypsilanti, MI 48197 / USA
E-mail: sales@lavisioninc.com / www.lavisioninc.com
Phone: (734) 485 - 0913 / Fax: (240) 465 - 4306

Obtaining Maps of Elastic Constants in a Multi-Material Component

S. Szyniszewski et al, Department of Engineering, Durham University, UK

This study assumed that the material is locally elastic, i.e. the stress-strain relationship can be described as:

$$\begin{bmatrix} \varepsilon^*_{xx,i} \\ \varepsilon^*_{yy,i} \\ \varepsilon^*_{zz,i} \\ \varepsilon^*_{yz,i} \\ \varepsilon^*_{zx,i} \\ \varepsilon^*_{xy,i} \end{bmatrix} = \frac{1}{E_i} \begin{bmatrix} 1 & -\nu_i & -\nu_i & 0 & 0 & 0 \\ -\nu_i & 1 & -\nu_i & 0 & 0 & 0 \\ -\nu_i & -\nu_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu_i \end{bmatrix} \begin{bmatrix} \tilde{\sigma}_{xx,i} \\ \tilde{\sigma}_{yy,i} \\ \tilde{\sigma}_{zz,i} \\ \tilde{\sigma}_{yz,i} \\ \tilde{\sigma}_{zx,i} \\ \tilde{\sigma}_{xy,i} \end{bmatrix}$$

where ν_i and E_i are unknown elastic constants at each element.

The algorithm was applied on an element-by-element basis to reconstruct the spatial distribution of elastic material properties. The resulting two-dimensional (x, y) stiffness maps describe variations in Young's modulus across the analyzed region as shown in Figure 5.

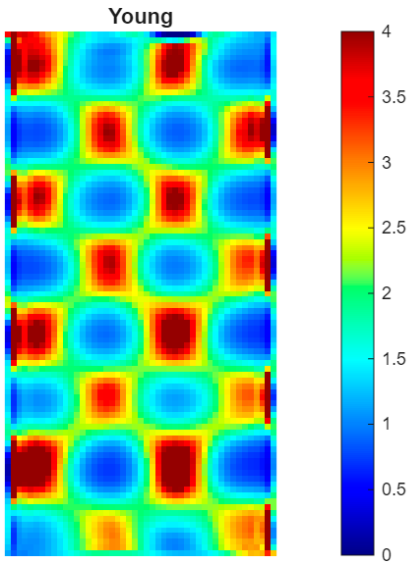


Figure 5: *Maps of as-manufactured Young's Modulus (MPa).*

A key advantage of this full-field approach is that it provides spatially resolved insight into material variability from the measured strain field under known loading conditions. At each iteration, the algorithm updates the local stresses and elastic constants and compares the resulting computational strain field with the experimentally measured DIC strain field (see Figure 5).

Convergence is defined by the relative root-mean-square (RMS) discrepancy between these two strain fields falling below 5%, indicating that the computationally reconstructed strain field agrees with the experimental measurements to within the prescribed tolerance. This convergence criterion is typically reached in fewer than 10 iterations.

The calculations are computationally efficient and can be performed rapidly on a standard personal computer. The computation of 8554 elastic parameters for the considered case, on a Dell XPS 9320 laptop, took 36 seconds. Representative convergence curves, showing the reduction in RMS discrepancy with successive iterations, are presented in Figure 6.

An important finding of this study is that the reconstructed material-property maps of the 3D-printed specimen did not reproduce the intended chessboard benchmark pattern precisely. Instead of the prescribed sharp step changes in stiffness between neighbouring regions, the experimentally reconstructed maps revealed gradual transitions, with the boundaries between the two material formulations appearing substantially more diffuse than anticipated. This behaviour could be attributed to diffusion of the cross-linking agent between neighbouring regions during manufacture, which effectively smooths the boundaries between the nominally discrete material formulations. The DIC processing parameters could also have an influence.

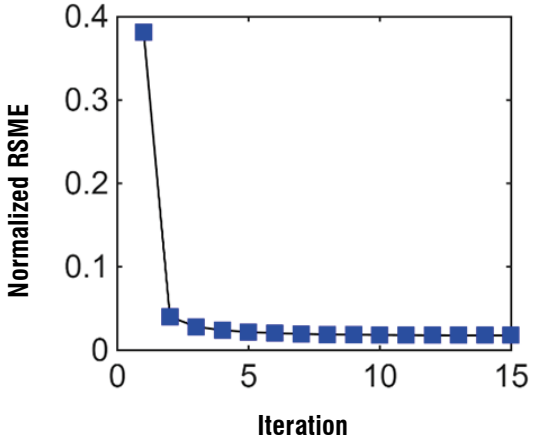


Figure 6: *Convergence of the normalized root mean square error of strain maps.*

LaVisionUK Ltd
2 Minton Place / Victoria Road
Bicester, Oxon / OX26 6QB / United Kingdom
E-Mail: sales@lavision.com / www.lavisionuk.com
Phone: +44-(0)-870-997-6532 / Fax: +44-(0)-870-762-6252

LaVision GmbH
Anna-Vandenhoeck-Ring 19
37081 Göttingen / Germany
E-Mail: info@lavision.com / www.lavision.com
Tel. +49-(0)551-9004-0 / Fax +49-(0)551-9004-100

LaVision Inc.
211 W. Michigan Ave. / Suite 100
Ypsilanti, MI 48197 / USA
E-mail: sales@lavisioninc.com / www.lavisioninc.com
Phone: (734) 485 - 0913 / Fax: (240) 465 - 4306

Obtaining Maps of Elastic Constants in a Multi-Material Component

S. Szyniszewski et al, Department of Engineering, Durham University, UK

Summary

This result highlights the potential for the as-manufactured properties of architectured multi-material components to deviate from their prescribed design distributions. Such deviations may arise from intrinsic limitations or previously uncharacterized features of additive manufacturing processes and may not necessarily be known a priori. Identifying these effects experimentally provides valuable information that can be incorporated into subsequent design iterations, allowing manufacturing-induced variability to be accounted for explicitly and improving the match between the intended and realized material-property distributions.

Furthermore, spatially resolved maps of both stress and elastic constants provide a means of quantifying material variability and identifying localized regions of reduced stiffness or other potential weaknesses. Such information could be particularly valuable for post-manufacturing inspection of bespoke components, enabling potentially critical material inhomogeneities to be detected before the component is incorporated into a finished part or more complex system.

Further information can be found in : Computation of Material Property Fields in Heterogeneous and Multi-Material Systems Using Inverse Gauss-Seidel Method, Guerra et al. https://dx.doi.org/10.2139/ssrn.5374720



Computation of material property fields in heterogeneous and multi-material systems using inverse Gauss-Seidel method

N. B. Guerra[a], C. Mo[b], S. J. C. Matthews[c], E. Street[d], J. R. Raney[b], S. R. Arwade[e], Lukasz Kaczmarczyk[f], L. C. M. Vieira Jr.[a], S. Szyniszewski[g,*]

[a]University of Campinas, Brazil
[b]Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, United States
[c]RINA Tech UK Ltd, United Kingdom
[d]Department of Aeronautics, University of Cambridge, United Kingdom
[e]University of Massachusetts at Amherst, United States
[f]University of Glasgow, United Kingdom
[g]University of Durham, United Kingdom

Abstract

The objective of this study is to compute spatial maps of elastic constants from strains measured using digital image correlation on multi-material, heterogeneous samples. Recent advances in multi-material 3D printing for biomedical and soft-robot applications enable the creation of components with complex architectures; however, there are few methods for their full-field characterisation, such as nanoindentation. Here, we formulate an energy functional for a numerical model, employing Lagrange multipliers to enforce agreement between experimental and numerical strains. We then apply a modified Gauss–Seidel block-iterative approach, which converges for a suite of 2D and 3D numerical benchmark cases. We also investigated the effect of experimental noise and applied the method to a multi-material experimental benchmark. The proposed solution strategy computed fields of elastic constants rapidly and with good engineering accuracy, establishing an efficient platform for the characterisation of 3D-printed, multi-material, heterogeneous components and their spatial statistics and variability. Future work may explore orthotropic, anisotropic and more nuanced models, as well as alternative numerical solution strategies.

Keywords: multi-material, heterogeneous material, inverse characterisation, full-field strains, elastic constants, Gauss-Seidel, inverse solution, full-field measurement,

*Corresponding author:
Email address: stefan.t.szyniszewski@durham.ac.uk (S. Szyniszewski)

This preprint research paper has not been peer reviewed. Electronic copy available at: https://ssrn.com/abstract=5374720

Preprint not peer reviewed



Dr. Stefan Szyniszewski
Associate Professor

LaVisionUK Ltd
2 Minton Place / Victoria Road
Bicester, Oxon / OX26 6QB / United Kingdom
E-Mail: sales@lavision.com / www.lavisionuk.com
Phone: +44-(0)-870-997-6532 / Fax: +44-(0)-870-762-6252

LaVision GmbH
Anna-Vandenhoeck-Ring 19
37081 Göttingen / Germany
E-Mail: info@lavision.com / www.lavision.com
Tel. +49-(0)551-9004-0 / Fax +49-(0)551-9004-100

LaVision Inc.
211 W. Michigan Ave. / Suite 100
Ypsilanti, MI 48197 / USA
E-mail: sales@lavisioninc.com / www.lavisioninc.com
Phone: (734) 485 - 0913 / Fax: (240) 465 - 4306